Two Techniques Of Tailoring A Global Harmonic Model: Operational Versus Model Approach Applied To The Egyptian Territory

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Abstract:

Due to the absence of the Egyptian terrestrial gravity data from the collected global data used for the solution of the international EGM96 harmonic model, this model was previously tailored to Egypt, using both the least-squares collocation (LSC) and the Stokes' integral methods. The relevant tailored models were called EGM96EGCT and EGM96EGIT and were estimated up to degree and order 599 and 650, respectively. In this paper, the two tailoring solutions, which are based on two different techniques, are compared according to their local performance in Egypt. The two models showed similar improvement in their fitness to the local Egyptian data, and consequently, they are considered equally capable of recovering the long-medium wavelength features of the gravity field in Egypt.

1 Introduction

By using a global harmonic model in local geoid solutions (Amin, 1983), the respective low frequency features would only be reliable, if the model contains local gravity data from the region under consideration (Shaker et. al, 1997). Concerning Egypt, the EGM96 harmonic model, as all other models, is claimed to lack in the terrestrial gravity data (Amin, 2002). Thus, the long wavelength features for the Egyptian territory cannot be optimally recovered from such global models, thus degrading the target precision of the local geoid solution.

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Hence, in a previous work by the authors (Amin et. al, 2002a&b), the EGM96 harmonic model was tailored to Egypt twice by the least-squares collocation (LSC) and the Stokes' integral techniques, based on the local Egyptian heterogeneous data, where tailoring a harmonic model to certain area means, that the data of that area is used to adjust or correct the harmonic model to fit that area. Particularly, in each case, corrections for the coefficients up to degree and order 360 were estimated and also the coefficients up to the maximum possible resolution were extracted. These two tailored models are denoted as EGM96EGCT and EGM96EGIT, up to degree and order 599 and 650, respectively. The aim of the current study is to compare the two tailored models, according to their local behavior in Egypt. The comparison showed a great agreement between the two solutions. Thus, regarding Egypt, both models practically possess the same improvement over the EGM96. This represents an implicit check on the reliability of both solutions.

2 Background

Tailoring a specific harmonic model to the local data in a certain region utilizes a local harmonic analysis scheme, which uses the respective data window as input. This procedure amounts to using the original model in a remove-restore procedure and predict an equivalent set of harmonic coefficients corrections up to the model's maximal degree (360 for EGM96) and coefficients of higher degrees up to the maximum possible resolution (Wenzel, 1998). In particular, if the input data are geoid undulations, N, then the EGM96 low frequency geoid part is removed to obtain the residual geoidal data δN,

$$\delta N = N - N_{EGM96}, \qquad (1)$$

where

$$N_{EGM96} = (GM/r\gamma) \sum_{n=0}^{360} (a/r)^n \sum_{m=0}^{n} (\bar{C}^{*}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\psi), \qquad (2)$$

where

ψ	the geocentric latitude,
λ	the geodetic longitude,
r	the geocentric radius,
$\gamma(\psi,r)$	the normal gravity induced by the WGS-84 reference ellipsoid,
GM	the Earth mass-Gravitational constant product consistent with the EGM96 coefficients,
a	the equatorial radius scale factor associated with theEGM96 model,
$\bar{C}^{*}_{nm}$	the EGM96 fully normalized spherical harmonic C-coefficients of degree n and order m, reduced for the even zonal harmonics of the WGS-84 reference ellipsoid,
$\bar{S}_{nm}$	the EGM96 fully normalized spherical harmonic S-coefficients of degree n and order m,
$\bar{P}_{nm}(\sin\psi)$	the fully normalized associated Legendre function of degree n and order m.

The obtained δN values are then used as input for the harmonic analysis algorithm (Eqs. (4) & (5)) to receive the harmonic coefficients corrections ΔC_{nm} & ΔS_{nm}. The coefficients' corrections (up to degree and order 360) are then restored back to the EGM96 relevant coefficients, in order to end up with the relevant tailored coefficients,

$$\bar{C}_{nm\ \text{tailored}} = \bar{C}_{nm\ \text{EGM96}} + \Delta C_{nm}, \quad (\text{for } n \leq 360)$$

$$\bar{S}_{nm\ \text{tailored}} = \bar{S}_{nm\ \text{EGM96}} + \Delta S_{nm}. \quad (\text{for } n \leq 360) \tag{3}$$

Logically, the resulting terms with $n > 360$ represent the coefficients themselves. Of course, the actual spectral content inherent into the data judges the maximum degree and order of the significant and reliable terms that could be extracted. The maximum degree and order depends also on the used technique for extracting them.

In general, spherical harmonic analysis (using gravimetric data as input) can be performed based on either the operational approach or the model approach. On one hand, the operational approach is primarily represented by the least-squares collocation technique. Also, the method of least-squares adjustment is classified as belonging to that approach. On the other hand, the model approach for harmonic analysis leans, analogous to Stokesian geoid determination, on the integral solution technique. In principle, both approaches for harmonic analysis base on the (unique) functional relationships between the various gravimetric quantities and the spherical harmonic coefficients.

Harmonic analysis by least-squares adjustment can be performed, using either the combined least-squares or the observation equation method. The least-squares formulation would have the (coefficients) corrections as parameters to be estimated, the input gravimetric data as observations and the original harmonic model terms as approximate parameters (Rapp, 1969). Remembering that the tailoring of EGM96 encountered the estimation of a huge number of terms, then using least-squares adjustment would have been impossible, since this requires a very large computational effort and storage capacity, which are far beyond a PC capability. In addition, the harmonic terms estimated by the least-squares adjustment model could suffer from the aliasing effects. This can be due to the fact that, in this case, the different estimated coefficients are forced to correlate, as a consequence of the least-squares adjustment algorithm (Desrochers, 1971). This correlation, in turn, affects the orthogonality among the different terms.

The above reasons were sufficient to choose the least-squares collocation and the integral solution technique as two alternatives to perform the local harmonic analysis. The geoidal height is regarded as the smoothest version of all anomalous quantities (Meissl, 1971). As the target was to estimate terms of low degree nature, the geoid was suggested to represent the most appropriate input data type to accomplish this task, using the two techniques. The original scattered point data was initially used to solve for a 5'x5' free air geoid grid, relative to the WGS-84 reference ellipsoid, covering the Egyptian territory (22°N≤φ≤32°N; 25°E≤λ≤36°E), based on EGM96. The geoid solution utilized the least-squares collocation technique, which had as input all the available heterogeneous gravimetric data. The values of the residual (δN) of the geoid at the grid nodes were then used for the tailoring process, using both approaches.

3 Harmonic analysis by collocation and integral techniques

Harmonic analysis by the (LSC) method has the advantage of its ability to deal with any heterogeneous scattered data format and the evaluation of the error estimates of the predicted harmonic coefficients. Moreover, on the contrary to least-squares adjustment, the harmonic coefficients are estimated separately without any restrictions on the size of the signal vector (Moritz, 1980 and Tscherning, 2001). In this manner, the estimated terms are not subject to the aliasing error, which is defined as the effect of the neglection of the estimation of the higher degree terms on the evaluated (lower degree) terms. Moreover, if a higher degree coefficient can no more be reliably determined, then collocation gives it a value close to zero (Moritz, 1973). Hence, a preliminary investigation was carried out in order to know the maximum obtainable resolution using the (LSC) principle, which was found to be up to degree and order 599, based on the heterogeneous point data. Particularly, further trials indicated a zero estimated value (with zero standard deviation) for any coefficient of higher degrees. This was also verified, using the 5'x5' residual geoid grid as input. Therefore, based on that expected resolution, it was decided to use a compatible coarser residual geoid grid (15'x15') as input data for harmonic analysis by collocation (Rapp, 1977). The resulting number of grid nodes was less than the scattered data number, thus reducing the computational burden, and at the same time a homogenous data coverage could be achieved. In the collocation harmonic analysis, the covariances between the gravimetric elements and the spherical harmonic coefficients were exploited to predict the coefficients (corrections) as well as their uncertainties, based on the input residual geoidal height grid along with its noise (Tscherning, 2001 and Amin et. al, 2002a). In this respect, the EGM96 model was firstly tailored to degree and order 360 (EGM96EGCR) and then was extended up to degree and order 599 (EGM96EGCT), using the following algorithm

$$(GM/R).\ \{\Delta C_{ij} :\ \Delta S_{ij}\} = C_{ij\,t}-(C_{tt} + E_{tt})^{-1}\ .\ l, \qquad (4a)$$

$$E_{ij\,ij} = C_{ij\,ij} - C_{ij\,t}.(C_{tt} + E_{tt})^{-1}\ .\ C_{ij\,t}^{T}, \qquad (4b)$$

with

R	the mean radius of the Earth,
$(GM/R). \{\Delta C_{ij} : \Delta S_{ij}\}$	the estimated signal (in potential units) ,
$C_{ij\,t}$	the cross-covariance vector between the signal and the (residual geoid) observations l,
C_{tt}	the covariance matrix of the (residual geoid) observations,
E_{tt}	the error variance-covariance matrix of the (residual geoid) observations,
l	the vector of (residual geoid) observations,
$E_{ij\,ij}$	the estimated error variance of the estimated signal,
$C_{ij\,ij}$	the signal variance.

On the other hand, harmonic analysis by the integral technique (the model approach) demanded a little computer time, compared to the operational approach. Using the input 5'x5' residual geoid grid (which contains 121 latitudes and 133 longitudes), a sequence of exploratory integral solutions was performed, in order to judge the maximum attainable resolution. This was accomplished by gradually increasing the maximum degree and order of the target model at arbitrary spectral intervals. Each solution was checked regarding its ability to further smooth the available gravity anomaly data set, via the removal of the respective harmonic model. It was found that beyond a maximum degree and order of about 650, the quality of the resulting tailored models degrades, in terms of their ability to smooth the gravity anomaly data. So, it was decided to consider the model tailored up to degree and order 650 as achieving the maximum resolution, which could be reliably extracted by this method, based on the original data density and coverage. This tailored model was called EGM96EGIT, while the model tailored to degree and order 360 only was denoted as EGM96EGIR (Amin et. al, 2002b). Finally, harmonic analysis via the model approach does not encounter aliasing effects (Desrochers, 1971), as the coefficients (corrections) are estimated separately, as follows

$$\Delta C_{nm} = (1 / (4\pi.GM)) \sum_{i=1}^{121} \sum_{j=1}^{133} r_i\, \gamma_i\, (r_i/a)^n\, \delta N\, (\varphi_i,\lambda_j).\cos m\lambda_j.\bar{P}_{nm}(\sin\psi_i).\cos\varphi_i\, .\ \Delta\varphi\, \Delta\lambda, \tag{5a}$$

$$\Delta S_{nm} = (1 / (4\pi.GM)) \sum_{i=1}^{121} \sum_{j=1}^{133} r_i\, \gamma_i\, (r_i/a)^n\, \delta N\, (\varphi_i,\lambda_j).\ \sin m\lambda_j\, .\bar{P}_{nm}(\sin\psi_i).\ \cos\varphi_i\, .\ \Delta\varphi\, \Delta\lambda, \tag{5b}$$

with

φ_i, λ_j	the geodetic coordinates of the running block center,
$\delta N\ (\varphi_i,\lambda_j)$	the relevant residual geoidal height,
$r_i\ (\varphi_i)$	the respective geocentric radius,
ψ_i	the relevant geocentric latitude,
$\Delta\varphi$, $\Delta\lambda$	the (equal) latitude and longitude grid intervals.
$\gamma_i(\varphi_i)$	the normal gravity at the running block center.

It should be emphasized that the (relatively small) disagreement between the resolutions of the two tailored models, EGM96EGCT and EGM96EGIT, could have occurred, due to the different theories relevant to the two prediction approaches. Another reason could be the approximation and technical discrepancies associated with the available harmonic analysis codes. This small mismatch will also manifest itself in the further results obtained from both approaches, as will be seen below.

4 Results

Table (1) shows the statistics of the residual 15´x 15´geoid grids, using the EGM96, EGM96EGCR and EGM96EGCT, respectively, based on the "full" input free air geoid. It is clear how much local information has been incorporated into the models tailored by collocation. On the other hand, Table (2) shows the same information for the residual 5´x 5´geoid grids, but regarding the models tailored by the model approach, namely, the EGM96EGIR and EGM96EDIT models. Of course, much local features have been also introduced to these tailored models. Regarding both tables, the refinement is implied by the great smoothness of the residuals, in terms of the mean and standard deviation. In brief, the EGM96EGCT and EGM96EGIT tailored models possess superior long to medium wavelength behaviors over the EGM96 model. Table (3) shows a similar result, regarding the spectral amount removed from the discrete gravity anomaly data, by the four tailored models, compared to the EGM96 harmonic model. Obviously both the EGM96EGCR and EGM96EGIR models have similar much improved behaviors, over the original model. The performance is further improved by the EGM96EGCT and EGM96EGIT tailored models.

Table (1): Statistics of the residual 15´x 15´ geoid grids using EGM96 and the models tailored by collocation (unit: meter)

	reference field	Mean	Std. Dev.	RMS	Min.	Max.
δN	EGM96	0.568	1.429	1.537	-3.924	5.655
	EGM96EGCR(360)	0.058	0.108	0.123	-0.480	0.650
	EGM96EGCT(599)	0.057	0.086	0.103	-0.310	0.497

Table (2): Statistics of the residual 5´x 5´ geoid grids using EGM96 and the models tailored by the integral technique (unit: meter)

	reference field	Mean	Std. Dev.	RMS	Min.	Max.
δN	EGM96	0.575	1.440	1.551	-3.924	5.719
	EGM96EGIR(360)	-0.011	0.077	0.078	-0.761	0.480
	EGM96EGIT(650)	-0.010	0.040	0.041	-0.335	0.238

Table (3) Statistics of the residual anomaly data sets from the discrete gravity anomaly data points (unit: mgal)

	reference field	Mean	Std. Dev.	RMS	Min.	Max.
$\delta\Delta g$	none (free air anomalies)	-5.916	29.142	29.725	-78.234	144.623
	EGM96	-1.839	23.344	23.407	-66.779	123.975
	EGM96EGCR(360)	-2.320	17.621	17.767	-96.610	61.265
	EGM96EGIR(360)	-0.455	17.531	17.530	-94.686	60.846
	EGM96EGCT(599)	-1.489	13.668	13.743	-75.136	53.820
	EGM96EGIT(650)	0.349	13.119	13.119	-68.017	50.902

Table (4) through (7) show the statistics of the (synthesized) gravity anomalies, geoidal heights, meridian and prime-vertical deflection components, respectively, computed from the EGM96 and the four tailored models at the nodes of a 15´x 15´ grid covering the Egyptian Territory. Clearly, the two tailoring approaches have resulted in equivocal changes, relative to EGM96. Considering the four gravimetric elements, the (signal) standard deviation and RMS values, pertaining to the four tailored models, are relatively greater than those relevant to the EGM96 model. This implies that, relative to the EGM96, the four tailored models have gained more detailed (or more rough) signal structure, as a result of the incorporation of the local gravity field features. Of course, the improvements are rather implied by the EGM96EGCT and EGM96EGIT models. Figure (1) through (3) show the contour maps of the gravity anomalies relevant to the EGM96, EGM96EGCT and EGM96EGIT models, respectively. While the EGM96 gives an anomaly scheme of a very crude long wavelength nature, both the EGM96EGCT and EGM96EGIT solutions equally give much more detailed structures. Particularly, the three maps show how efficiently the actual long-medium wavelength features, which are consistent with the input data, have been launched into the tailored models, irrespective of which harmonic analysis approach has been followed. The same remarks can be deduced for geoidal heights from Figure (4) through (6), which show the respective geoidal contour maps.

Table (4): Comparison among the 15´x 15´ gravity anomalies computed from EGM96 and the tailored models
(unit: mgal)

Δg	reference field	Mean	Std. Dev.	RMS	Min.	Max.
	EGM96	5.370	24.727	25.297	-130.589	153.238
	EGM96EGCR(360)	6.638	34.725	35.345	-171.108	133.188
	EGM96EGIR(360)	3.105	35.143	35.270	-179.862	133.562
	EGM96EGCT(599)	6.678	35.218	35.836	-173.227	133.937
	EGM96EGIT(650)	2.604	35.774	35.859	-177.025	143.749

Table (5): Comparison among the 15´x 15´geoidal heights computed from EGM96 and the tailored models (unit: meter)

	reference field	Mean	Std. Dev.	RMS	Min.	Max.
N	EGM96	13.817	2.743	14.087	7.532	21.124
	EGM96EGCR(360)	14.327	3.317	14.706	6.869	22.032
	EGM96EGIR(360)	14.401	3.340	14.783	6.715	22.243
	EGM96EGCT(599)	14.328	3.318	14.706	6.825	22.053
	EGM96EGIT(650)	14.400	3.341	14.782	6.757	22.208

Table (6): Comparison among the 15´x 15´meridian deflection components computed from EGM96 and the tailored models (ξ unit: arc-second)

	reference field	Mean	Std. Dev.	RMS	Min.	Max.
ξ	EGM96	-0.714	3.903	3.967	-16.621	22.849
	EGM96EGCR(360)	-0.997	5.285	5.377	-22.022	23.437
	EGM96EGIR(360)	-0.911	5.441	5.515	-22.523	25.072
	EGM96EGCT(599)	-0.995	5.366	5.456	-23.117	24.303
	EGM96EGIT(650)	-0.858	5.614	5.678	-23.480	27.920

Table (7): Comparison among the 15´x 15´prime-vertical deflection components computed from EGM96 and the tailored models (η unit: arc-second)

	reference field	Mean	Std. Dev.	RMS	Min.	Max.
η	EGM96	0.539	4.196	4.229	-33.231	17.141
	EGM96EGCR(360)	0.553	5.552	5.578	-33.337	22.250
	EGM96EGIR(360)	0.564	5.621	5.648	-33.312	22.758
	EGM96EGCT(599)	0.554	5.607	5.633	-33.038	22.635
	EGM96EGIT(650)	0.566	5.752	5.778	-32.381	22.530

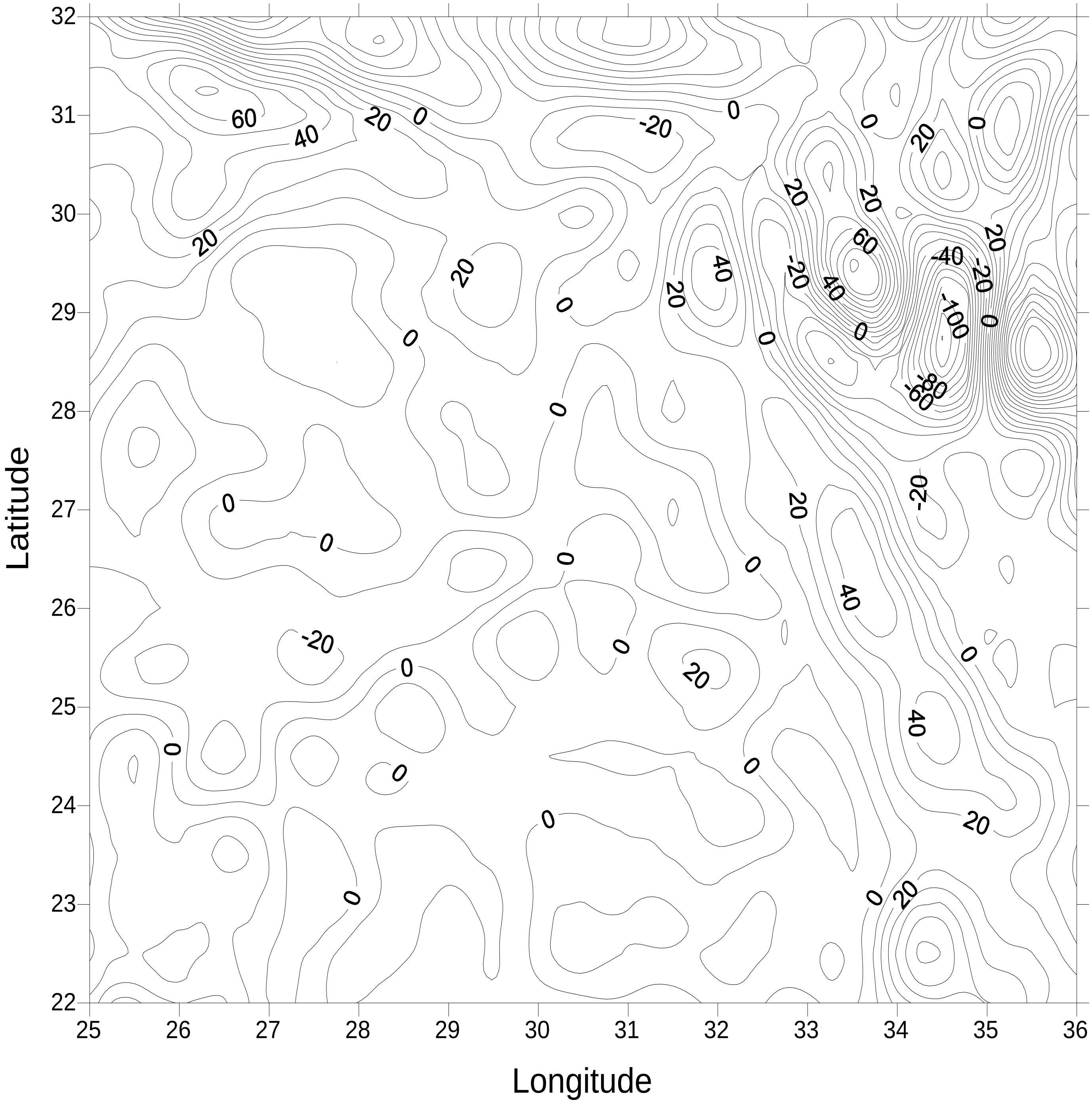

Figure (1): Contour map of EGM96 gravity anomalies
(Interval: 10 mgal)

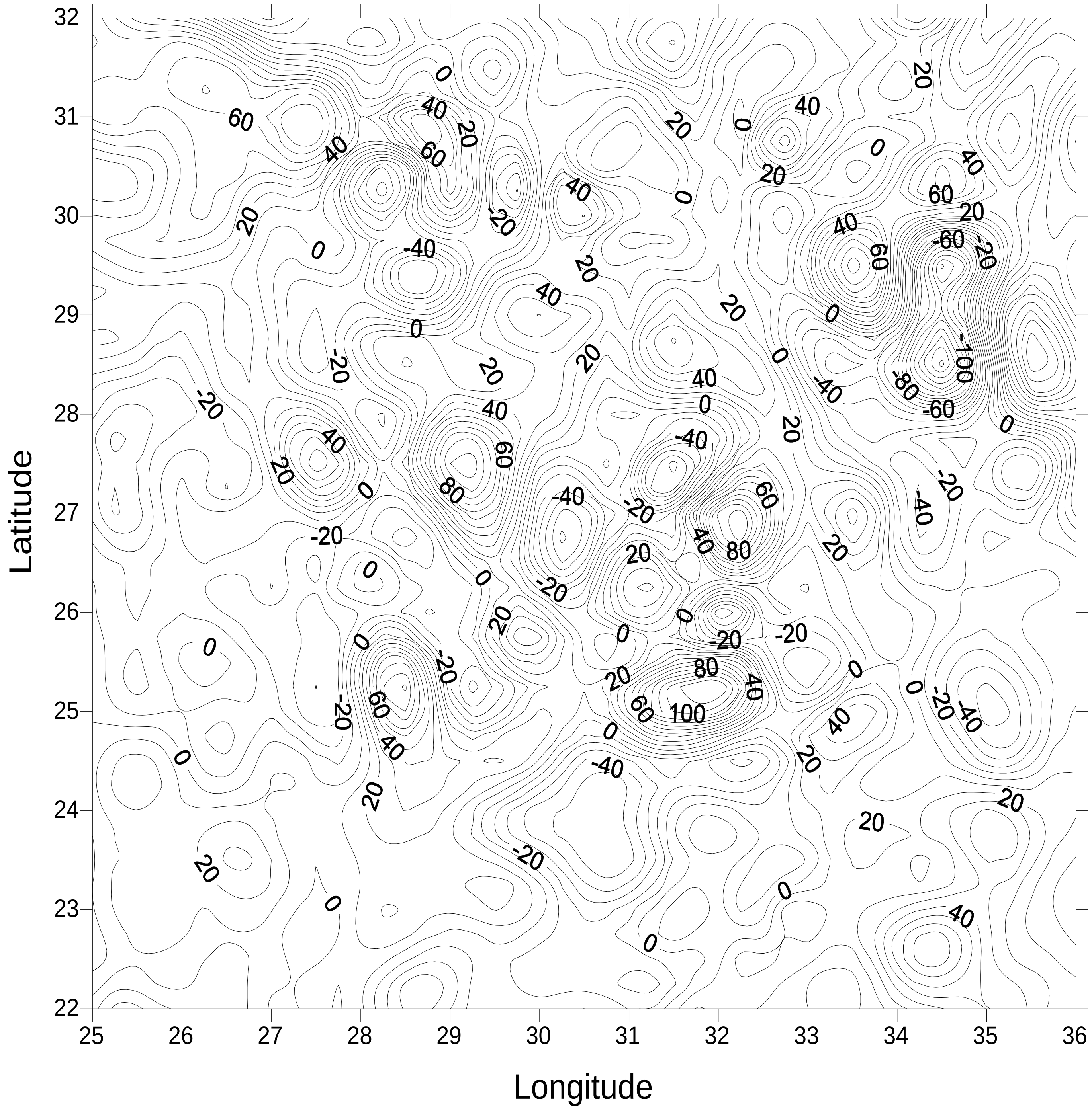

Figure (2): Contour map of EGM96EGCT gravity anomalies (Interval: 10 mgal)

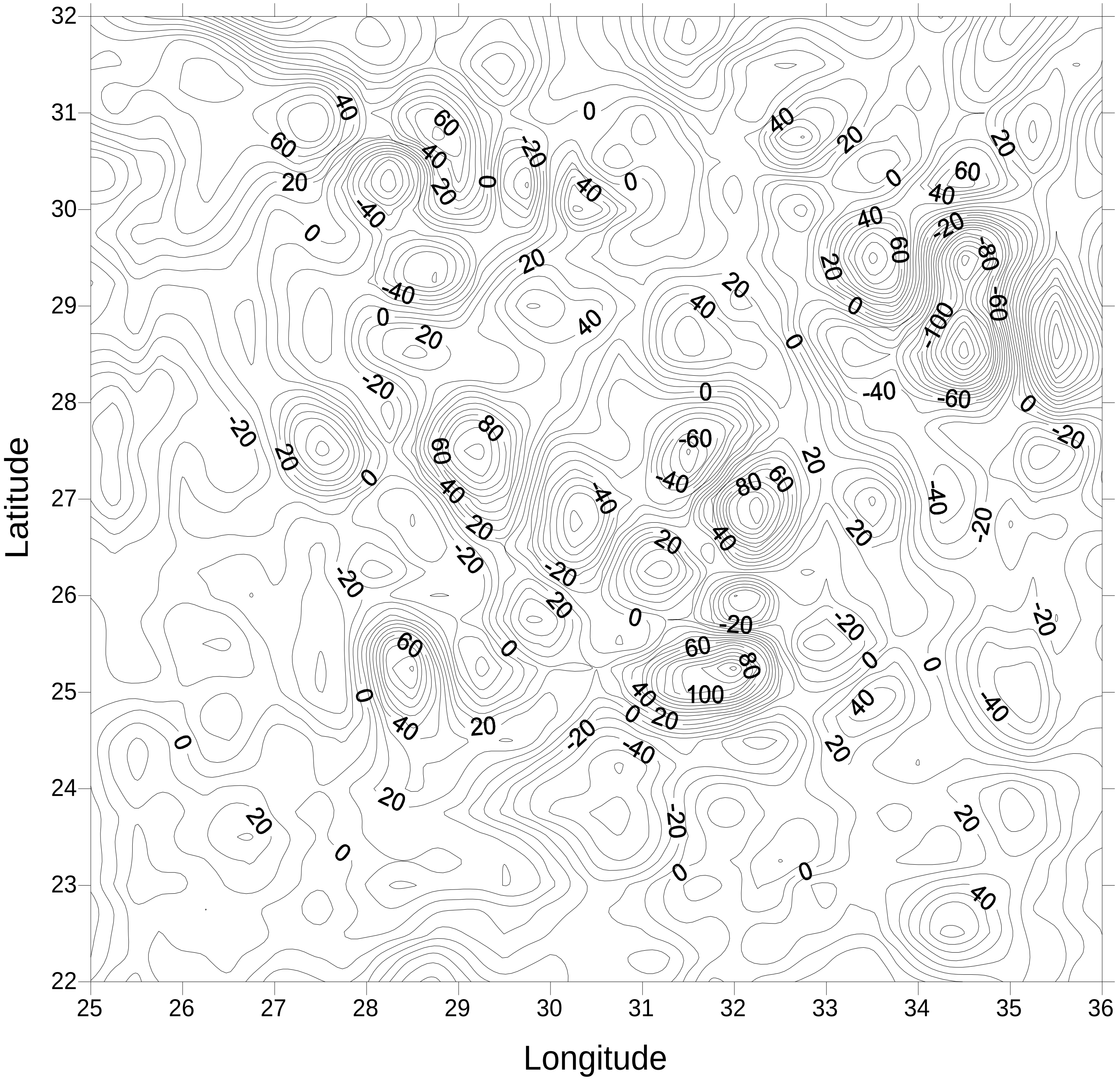

Figure (3): Contour map of EGM96EGIT gravity anomalies
(Interval: 10 mgal)

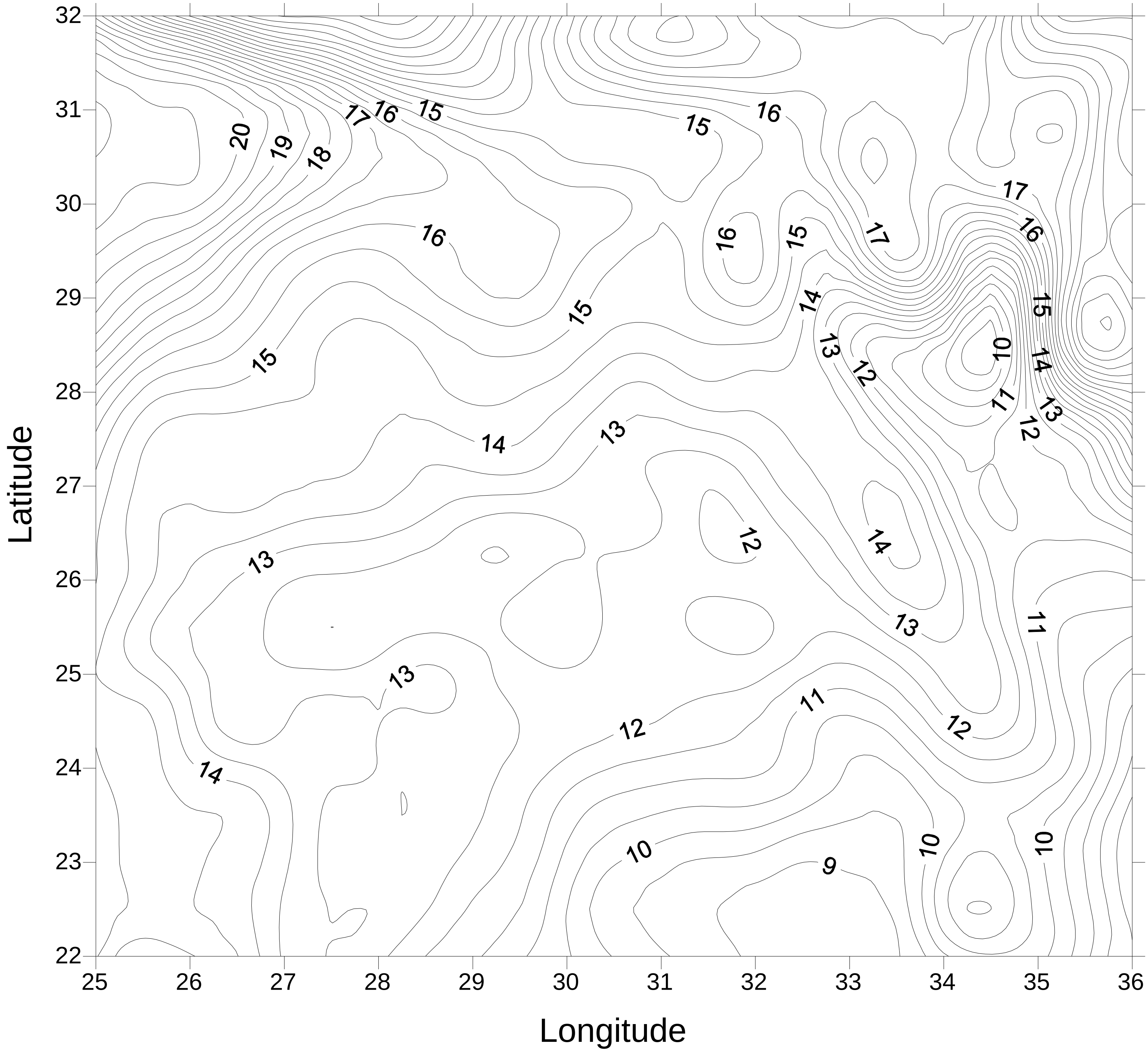

Figure (4): Contour map of EGM96 geoidal heights (Interval: 0.5 m)

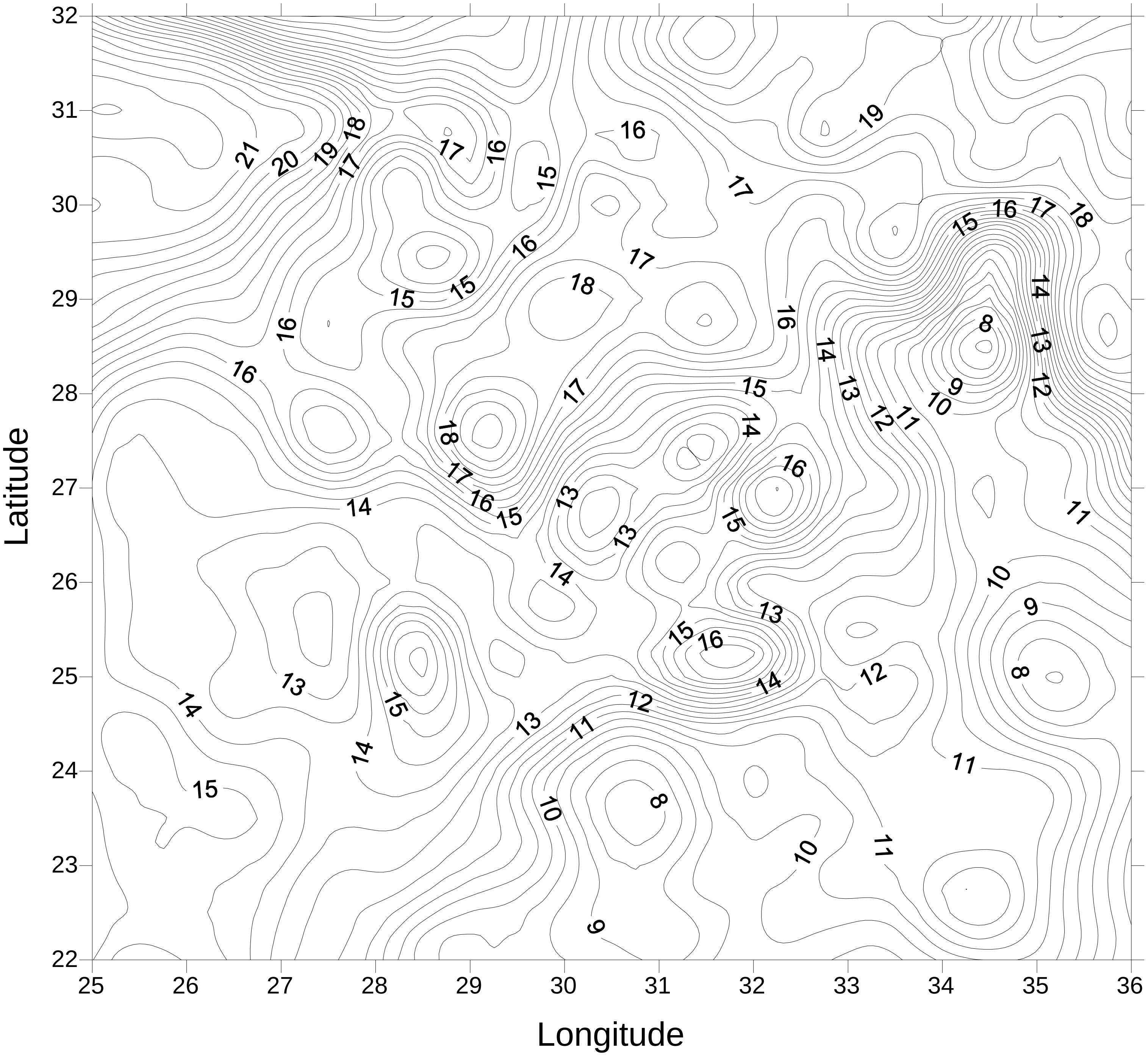

Figure (5): Contour map of EGM96EGCT geoidal heights
(Interval: 0.5 m)

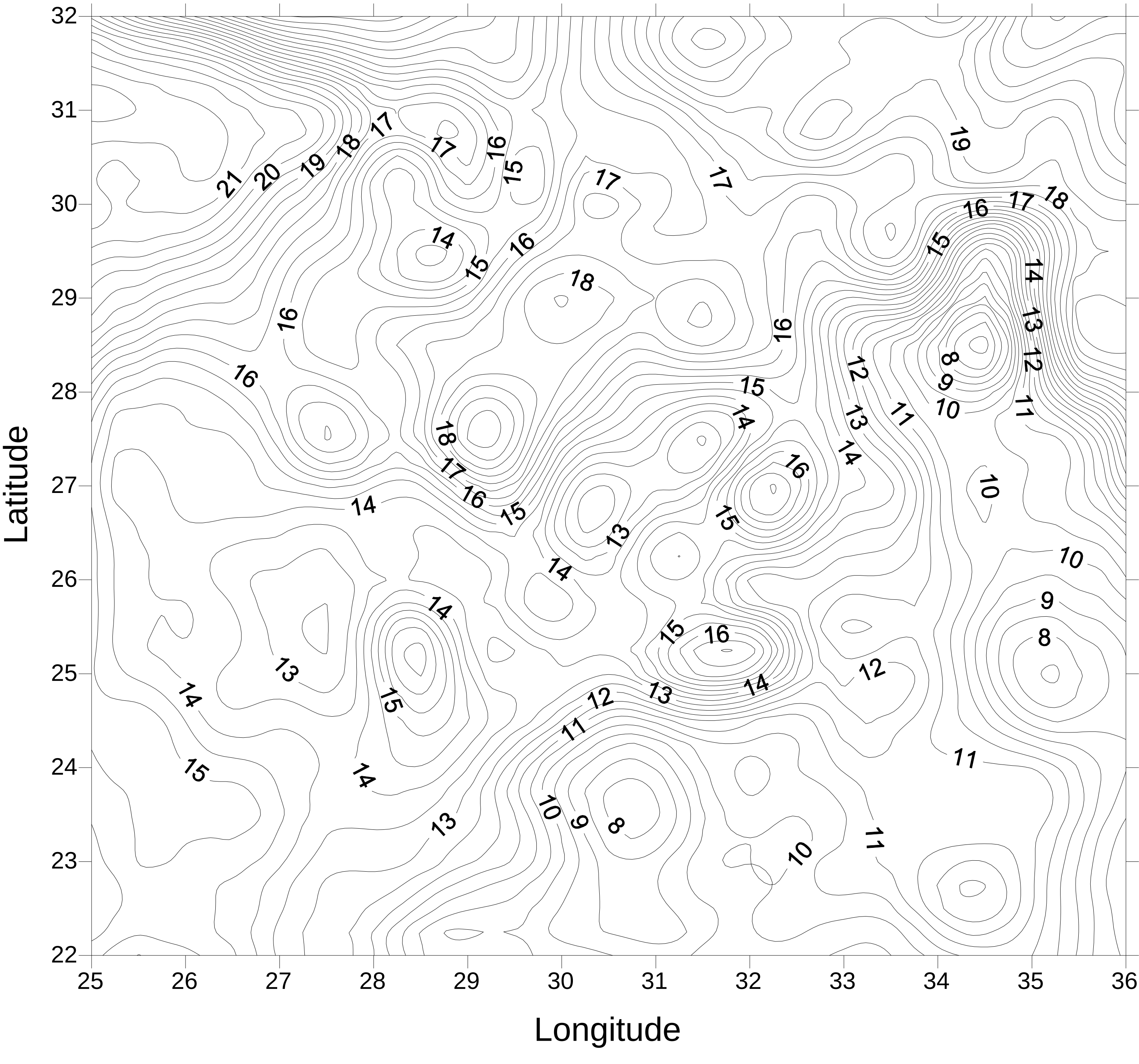

Figure (6): Contour map of EGM96EGIT geoidal heights (Interval: 0.5 m)

5 Concluding remarks

It can be concluded that the EGM96EGCT and EGM96EGIT models, sound almost the same, regarding the achieved local improvement over the EGM96 field. Of course, small differences exist among the relevant results, due to the respective theoretical bases as well as the discrepancies between the computational software codes. Nevertheless, the reliability of both tailored models has been verified. Hence, both tailored models are equally recommended to model the low-medium spectral features over Egypt in an efficient manner.

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